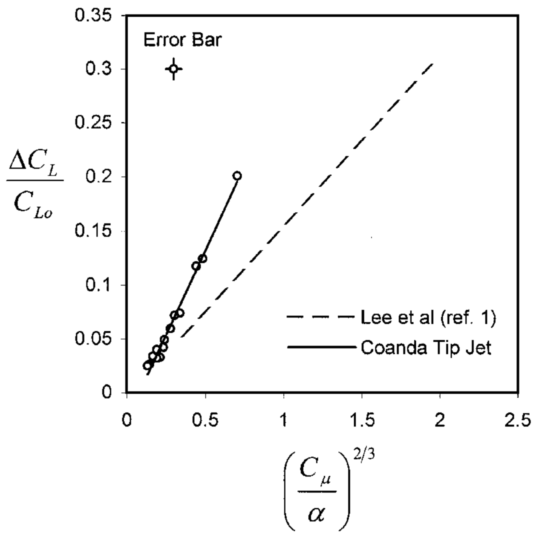

Fig. 2 Lift enhancement due to blowing.

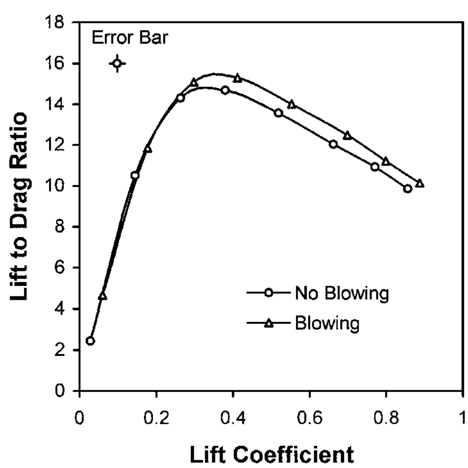

Fig. 3 Aerodynamic efficiency due to blowing at C_μ = 0.01.

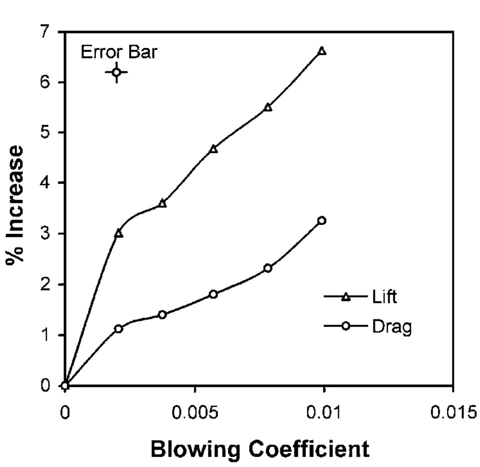

Fig. 4 Percentage increase in lift and drag due to blowing at α = 8 deg.

of blowing have minimal effect on the overall lift-to-drag ratio. The enhanced performance appears to be fairly constant above a C_L of 0.4.

Effects of Varying Blowing Coefficient

The effect of blowing on the lift and drag of the wing was studied for five blowing coefficients at a constant angle of attack. Over the values tested, the enhancement was observed to be approximately linear except for the sharper increase in both lift and drag coefficients with the initial introduction of blowing. Figure 4 shows the percentage increase in the lift and drag over the range tested.

Conclusions

Pneumatic blowing through a Coanda tip jet was investigated as a form of increasing the lift-to-drag ratio of a rectangular wing. The scheme was compared to another investigation and shown to have better lift enhancement characteristics. A significant increase in the lift-to-drag ratio in the higher C_L range was produced but little gain was experienced over the lower lift coefficients. Such a technique has the potential of improving a wing's lifting efficiency, particularly during the takeoff and landing phases of flight.

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Divergence of an Inflated Wing

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Introduction

A WING design with inflated tubular spars has unusual structural characteristics that will not accept the classical aeroelastic divergence formulation. This wing construction is particularly well suited for gun-launched observation vehicles and similar systems, where both weight and packaged volume must be minimized while maintaining aerodynamic efficiency of the deployed wing. As shown in Fig. 1, the wing is made up of several inflated woven fabric spars with circular cross section and impervious plastic lining. The wing is covered by a fabric skin. Flexible foam fills the volume between the spars and skin to provide the airfoil shape. The spars are connected at the root to the fuselage and at the tip to a rigid cap. Although the fabric skin does prevent chordwise distortion, the spars are not structurally connected either to the skin or to each other. At a wing section, then, resistance to twist must come from shear reactions rather than torsional stress. At root and tip, however, both torsional moment and shear are applied to each spar. Bending moments are, of course, exerted on the spars at the root, but not at the tip, where all external moments are zero.

The classical second-order differential equation for wing divergence[1,2] is not applicable here because, as will be seen later, reaction to torque at a wing section is proportional to the third derivative of the torsional deflection, rather than the first derivative. In what follows, the effective section torsional stiffness is first quantified. The differential equation for divergence of a constant-chord unswept wing is then derived, and divergence boundaries are calculated. Results indicate that the structure is very resistant to divergence, even though the spars only react torsionally through the end cap.

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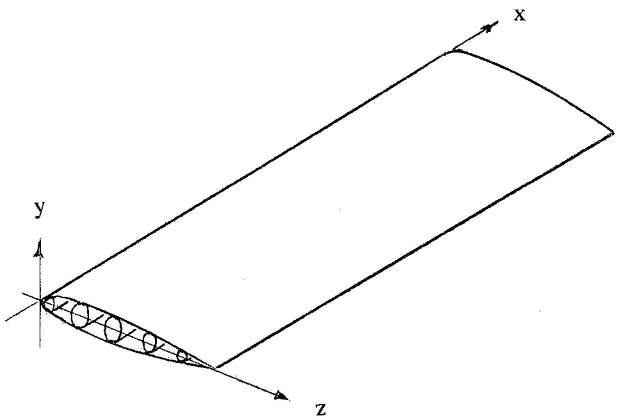

Fig. 1 Wing with inflated spars, coordinate system.

Torsional Stiffness due to Spar Bending

Consider a wing with N tubular spars, as shown in Fig. 1. If a pure torque τ is applied outboard of spanwise coordinate x, about x, the shears V_i in the spars must provide the reaction to it, such that

$$\tau(x) = \sum_{i=1}^{N} z_i V_i \tag{1}$$

Because no net force is exerted, it follows that

$$\sum_{i=1}^{N} V_i = 0 \tag{2}$$

Now, if δ_i is the deflection of spar i,

$$V_i = EI_i \frac{d^3\delta_i}{dx^3} \equiv EI_i \eta_i \tag{3}$$

where EI_i is the equivalent spar bending stiffness. Because the fabric skin prevents chordwise deformation, if θ is the section angle to the x–z plane, then

$$\theta = \frac{\delta_1 - \delta_2}{z_2 - z_1} = \frac{\delta_2 - \delta_3}{z_3 - z_2} = \cdots = \frac{\delta_{N-1} - \delta_N}{z_N - z_{N-1}}$$

Differentiating the preceding equation yields

$$\frac{d^3\theta}{dx^3} = \frac{\eta_1 - \eta_2}{z_2 - z_1} = \frac{\eta_2 - \eta_3}{z_3 - z_2} = \cdots = \frac{\eta_{N-1} - \eta_N}{z_N - z_{N-1}} \tag{4}$$

With V_i and η_i related by Eq. (3), Eqs. (1), (2), and (4) constitute N linear equations for the unknown shears. If the results, which are proportional to τ, are substituted in Eq. (4), it follows that

$$\tau(x) = k_b \frac{d^3\theta}{dx^3} \tag{5}$$

To complete the characterization of the wing torsional resistance, the location of the elastic axis must be defined. This can be extracted from the solution by computing the η_i values from Eq. (4). Because, from Eq. (4), they vary linearly with z, the elastic axis is located by a change in sign. That is, if η_j and η_{j+1} differ in sign, the elastic axis coordinate is given by

$$z_{EA} = \frac{\eta_j z_{j+1} - \eta_{j+1} z_j}{\eta_j - \eta_{j+1}} \tag{6}$$

Divergence Formulation

If q is dynamic pressure, c is chord, ec is the distance aft of the center of pressure of the elastic axis, and a is the section lift coefficient derivative with respect to angle of attack, the aerodynamic torque per unit span is given by

$$\frac{d\tau}{dx} = qec^2 a\theta$$

If the wing has constant chord and is not swept, the torque at spanwise coordinate x is given by

$$\tau(x) = k_b \frac{d^3\theta}{dx^3} = qec^2 a \int_x^s \theta(x')\, dx' + \tau_v \tag{7}$$

where s is semispan and τ_v is the shear torque exerted by the end cap. By the differentiation of Eq. (7),

$$k_b \frac{d^4\theta}{dx^4} + qec^2 a\theta = 0 \tag{8}$$

The end conditions are

$$\theta = \frac{d\theta}{dx} = 0, \qquad x = 0$$

$$\frac{d^2\theta}{dx^2} = 0, \qquad \frac{d^3\theta}{dx^3} = \frac{\tau_v}{k_b}, \qquad x = s$$

To specify τ_v, the equilibrium of the end cap must be considered. The end cap is subjected not only to the reaction shear torque $-\tau_v$, but also to the torsional reaction of the individual spars. If GJ is the sum of the spar torsional stiffnesses, it follows that

$$\tau_v = -(GJ/s)\theta(s) \tag{9}$$

because all of the spars are twisted an equal amount $\theta(s)$ at the wing tip.

Let $\xi = x/s$, and define β according to

$$\beta^4 = \frac{qec^2 s^4 a}{4k_b}$$

so that Eq. (8) can be written in dimensionless form as

$$\frac{d^4\theta}{d\xi^4} + 4\beta^4\theta = 0 \tag{10}$$

The general solution of this equation is

$$\theta = \sin\beta\xi(C_1 \sinh\beta\xi + C_2\cosh\beta\xi)$$
$$+ \cos\beta\xi(C_3\sinh\beta\xi + C_4\cosh\beta\xi)$$

By applying the four end conditions, a set of linear equations for C_1–C_4 is obtained. The condition for divergence is that the determinant of the coefficients of this set of equations vanish. Specifically, it was found that

$$\Delta = 0 = b_{11}b_{22} - b_{21}b_{12} \tag{11}$$

where

$$b_{11} = \cos\beta, \qquad b_{12} = \sin\beta + \cos\beta\tanh\beta$$

$$b_{21} = (2\beta^3/\gamma)(\cos\beta\tanh\beta - \sin\beta) + \sin\beta\tanh\beta$$

$$b_{22} = (4\beta^3/\gamma)\cos\beta + \sin\beta - \cos\beta\tanh\beta$$

where

$$\gamma = GJs^2/k_b$$

The smallest value of β satisfying Eq. (11) defines the divergence boundary.

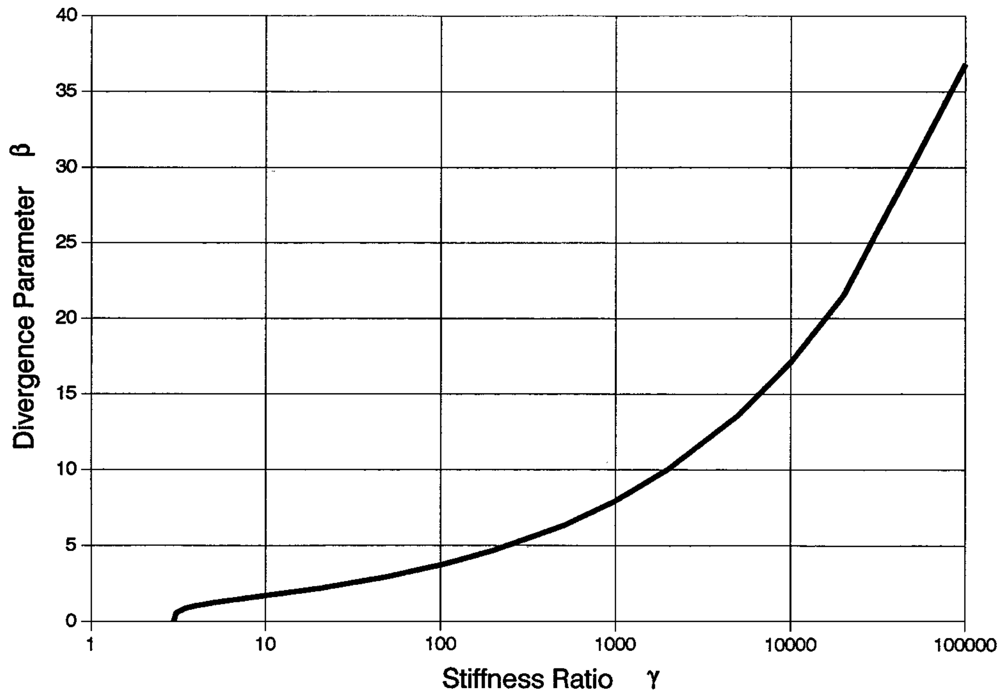

Fig. 2 Smallest root of the determinantal equation as a function of γ.

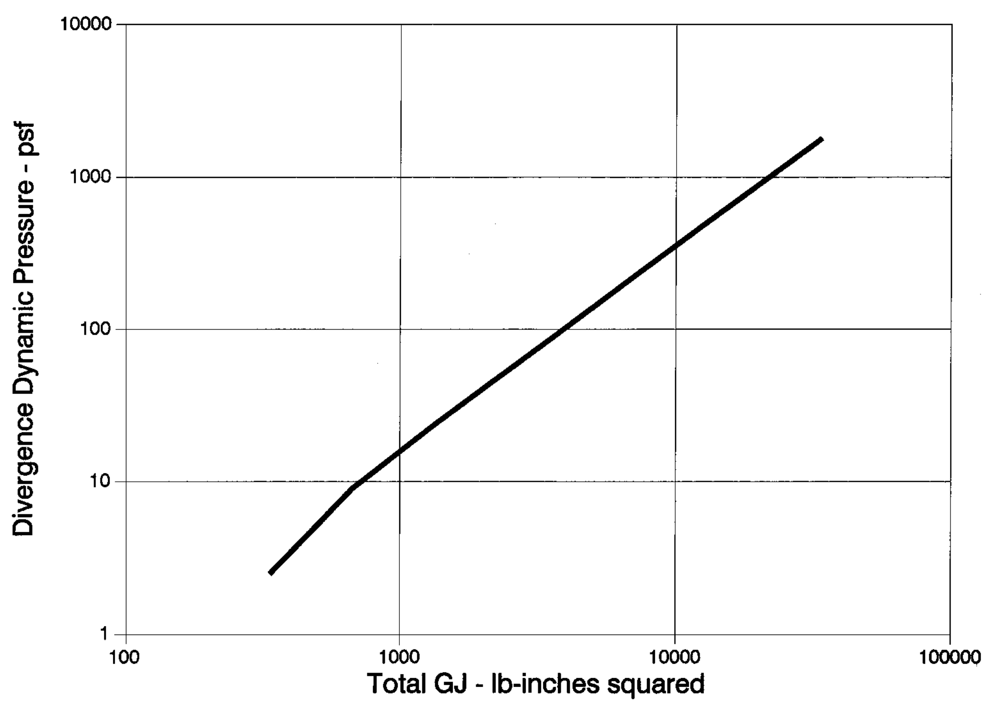

Fig. 3 Divergence boundary for a five-spar design.

Divergence Boundary

The smallest root of Eq. (11) was found numerically as a function of γ with the result shown in Fig. 2. For γ greater than about 100, the smallest root is simply obtained from $\beta^3 = \gamma/2$, the hyperbolic tangent then being very nearly equal to 1.

The spar torsional stiffness required to prevent divergence for a specific design was calculated using the preceding results. A design with five spars was analyzed. Given individual spar bending stiffnesses and positions, the torsional stiffness due to bending was found:

$$k_b = 2.90 \text{ lb-ft}^4$$

With a chord c of 0.594 ft, the elastic axis was found to be $0.13c$ aft of the quarter-chord. Semispan s is 2.5 ft and lift coefficient derivative was taken to be 5.73. Divergence dynamic pressure is plotted against collective spar torsional stiffness GJ in Fig. 3. Because the spar weave is estimated to provide a GJ value on the order of 800×10^6 lb-in.^{2}, this design is highly resistant to aeroelastic divergence at subsonic speeds.

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